Half-Angle Formulas

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\cos 2x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$$

Replace x with $\frac{x}{2}$

$$\boxed{\cos^2 \frac{x}{2}} = \frac{1+\cos 2\left(\frac{x}{2}\right)}{2} = \boxed{\frac{1+\cos x}{2}}$$

$$\boxed{\sin^2 \frac{x}{2}} = \frac{1-\cos 2\left(\frac{x}{2}\right)}{2} = \boxed{\frac{1-\cos x}{2}}$$

$$\boxed{\tan^2 \frac{x}{2}} = \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}} = \frac{\frac{1-\cos x}{2}}{\frac{1+\cos x}{2}} = \boxed{\frac{1-\cos x}{1+\cos x}}$$

Find exact value of $\sin\frac{\pi}{8}$

$$\frac{\pi}{8} = \frac{\frac{\pi}{4}}{2}$$

$$\sin^2\frac{\pi}{8} = \sin^2\frac{\frac{\pi}{4}}{2} = \frac{1-\cos\frac{\pi}{4}}{2} = \frac{1-\frac{\sqrt{2}}{2}}{2}$$

Let $x = \frac{\pi}{4}$

LCD = 2

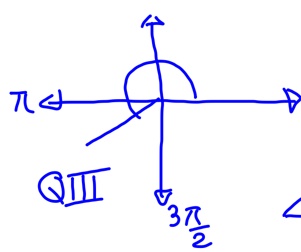
$$= \frac{2-\sqrt{2}}{4}$$

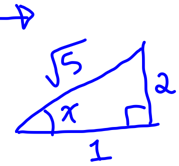
$$\sin\frac{\pi}{8} = \sqrt{\sin^2\frac{\pi}{8}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$$

Given $\tan x = 2$, $\pi < x < \frac{3\pi}{2}$, find $\tan\frac{x}{2}$

$$\tan^2\frac{x}{2} = \frac{1-\cos x}{1+\cos x}$$





$$\tan^2\frac{x}{2} = \frac{1-\left(\frac{-1}{\sqrt{5}}\right)}{1+\left(\frac{-1}{\sqrt{5}}\right)}$$

$$\cos x = \frac{-1}{\sqrt{5}}$$

$$= \frac{\sqrt{5}+1}{\sqrt{5}-1}\cdot\frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-1^2}$$

$$= \frac{(\sqrt{5}+1)^2}{4}$$

$$\tan^2\frac{x}{2} = \frac{(\sqrt{5}+1)^2}{4} \qquad \tan\frac{x}{2} = \pm\sqrt{\frac{(\sqrt{5}+1)^2}{4}} = \pm\frac{\sqrt{5}+1}{2}$$

$$\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

$\frac{x}{2}$ in QII

$$\boxed{\tan\frac{x}{2} = -\frac{\sqrt{5}+1}{2}}$$

$\sin\frac{7\pi}{12} = \frac{\sqrt{2}+\sqrt{6}}{4}$, $\sin\frac{7\pi}{12} = \frac{1}{2}\sqrt{2+\sqrt{3}}$

Show these two answers are equal.

$$\left[\frac{\sqrt{2}+\sqrt{6}}{4}\right]^2 = \left[\frac{\sqrt{2}+\sqrt{2}\sqrt{3}}{4}\right]^2 = \left[\frac{\sqrt{2}(1+\sqrt{3})}{4}\right]^2$$

$$= \frac{2(1+\sqrt{3})^2}{16} = \frac{1}{8}(1+2\sqrt{3}+3)$$

$$= \frac{1}{8}(4+2\sqrt{3})$$

$$= \frac{2(2+\sqrt{3})}{8}$$

$$= \frac{2+\sqrt{3}}{4}$$

NOW

$$\sqrt{\left[\frac{\sqrt{2}+\sqrt{6}}{4}\right]^2} = \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$