Half-Angle Formulas

$$
\begin{aligned}
& \cos 2 x=\cos ^{2} x-\sin ^{2} x \\
& \cos 2 x=2 \cos ^{2} x-1 \Rightarrow \cos ^{2} x=\frac{1+\cos 2 x}{2} \\
& \cos 2 x=1-2 \sin ^{2} x \Rightarrow \sin ^{2} x=\frac{1-\cos 2 x}{2}
\end{aligned}
$$

Replace $x$ with $\frac{x}{2}$

$$
\begin{aligned}
& \cos ^{2} \frac{x}{2}=\frac{1+\cos 2\left(\frac{x}{2}\right)}{2}=\frac{1+\cos x}{2} \\
& \sin ^{2} \frac{x}{2}=\frac{1-\cos 2\left(\frac{x}{2}\right)}{2}=\frac{1-\cos x}{2} \\
& \tan ^{2} \frac{x}{2}=\frac{\sin ^{2} \frac{x}{2}}{\cos ^{2} \frac{x}{2}}=\frac{\frac{1-\cos x}{2}}{\frac{1+\cos x}{2}}=\frac{1-\cos x}{1+\cos x}
\end{aligned}
$$

find exact value of $\sin \frac{\pi}{8}$

$$
\begin{aligned}
& \frac{\pi}{8}=\frac{\frac{\pi}{4}}{2} \\
& \begin{aligned}
& \operatorname{Sin}^{2} \frac{\pi}{8}=\sin ^{2} \frac{\pi}{4} \\
& \text { Let } x=\frac{\pi}{4} \frac{1-\cos \frac{\pi}{4}}{2}=\frac{1-\frac{\sqrt{2}}{2}}{2} \\
& \operatorname{LCD}=2 \\
& \sin \frac{\pi}{8}=\sqrt{\sin ^{2} \frac{\pi}{8}} \\
&=\sqrt{\frac{2-\sqrt{2}}{4}} \\
&=\frac{2-\sqrt{2}}{\sqrt{4}}=\frac{\sqrt{2-\sqrt{2}}}{2}
\end{aligned}
\end{aligned}
$$

Given $\tan x=2, \pi<x<\frac{3 \pi}{2}$, find $\tan \frac{x}{2}$

$$
\begin{aligned}
& \tan ^{2} \frac{x}{2}=\frac{1-\cos x}{1+\cos x} \\
& \begin{aligned}
& \tan ^{2} \frac{x}{2}=\frac{1-\left(\frac{-1}{\sqrt{5}}\right)}{1+\left(\frac{-1}{\sqrt{5}}\right)} \\
&=\frac{\sqrt{5}+1}{\sqrt{5}-1} \cdot \frac{\sqrt{5}+1}{\sqrt{5}+1}=\frac{(\sqrt{5}+1)^{2}}{(\sqrt{5})^{2}-1^{2}} \\
& \cos x=\frac{-1}{\sqrt{5}} \\
&=\frac{(\sqrt{5}+1)^{2}}{4} \\
& \tan ^{2} \frac{x}{2}=\frac{(\sqrt{5}+1)^{2}}{4} \quad \tan \frac{x}{2}= \pm \sqrt{\frac{(\sqrt{5}+1)^{2}}{4}}= \pm \frac{\sqrt{5}+1}{2} \\
& \pi<x<\frac{3 \pi}{2} \Rightarrow \frac{\pi}{2}<\frac{x}{2}<\frac{3 \pi}{4} \quad \frac{x}{2} \text { in QII } \quad \tan \frac{x}{2}=-\frac{\sqrt{5}+1}{2}
\end{aligned}
\end{aligned}
$$

$$
\sin \frac{7 \pi}{12}=\frac{\sqrt{2}+\sqrt{6}}{4}, \sin \frac{7 \pi}{12}=\frac{1}{2} \sqrt{2+\sqrt{3}}
$$

Show these two answers are equal.

$$
\left.\begin{array}{l}
\text { Show these two answers are } \begin{array}{rl}
{\left[\frac{\sqrt{2}+\sqrt{6}}{4}\right]^{2}} & =\left[\frac{\sqrt{2}+\sqrt{2} \sqrt{3}}{4}\right]^{2}
\end{array}=\left[\frac{\sqrt{2}(1+\sqrt{3})}{4}\right]^{2} \\
\\
=\frac{2(1+\sqrt{3})^{2}}{16}=\frac{1}{8}(1+2 \sqrt{3}+3) \\
\end{array}=\frac{1}{8}(4+2 \sqrt{3})\right]=\frac{2(2+\sqrt{3})}{8} . \begin{aligned}
& \sqrt{\left[\frac{\sqrt{2}+\sqrt{6}}{4}\right]^{2}}=\sqrt{\frac{2+\sqrt{3}}{4}} \\
&=\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}}=\frac{\sqrt{2+\sqrt{3}}}{2}=\frac{2+\sqrt{3}}{4}
\end{aligned}
$$

